Reflections on learning mathematics

Increasingly, the cultural role of mathematics is being recognized, and mathematics education as a provider of this culture. Like other disciplines, mathematics is a continually expanding and increasingly complex field, which also brings consequences for mathematics education. Although it has traditionally been present in education, it can and should be taught with procedures different from the traditional, such as those raised in this article.

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When we analyse the poor results of the boys and girls from second grade on the Student Census Evaluation (ECE) in recent years, particularly those obtained in tests of mathematics, we are obliged to think about it and demand serious action to generate an improvement in the country.

Is this situation the responsibility of the curriculum, teachers, and society as a whole? We should highlight that we are talking about the results of children in second grade and not students who have already spent a long stretch in the educational system and may have been subjected to much more complex demands.

The results of these tests in the area of mathematics are significantly lower than those of communication in almost all cases, even in schools where the overall results are not so worrying. This contains much to think regarding the currently status of mathematics in Peru. And when we speak of mathematics we must refer not only to the formal learning that occurs in classrooms, but also to the presence, evaluation and use of mathematics in general in our society.

Considering that mathematics is a fundamental part of the culture and development of humankind, which provides the models we use practically in all branches of human knowledge and allows us to understand and use the patterns that already exist in nature and the world around us, it is impressive how little most people really know them. Also, it is interesting the attitude facing this ignorance we usually find: resignation and little motivation to do something about it, as if it really did not

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matter, to the point that this attitude is often transmitted from parents to children as a legacy.

And, on the other hand, although mathematics is everywhere and used every day without knowing it, it is a science in itself with significant cognitive demands, and its learning is a permanent challenge.

Therefore, the learning of mathematics in school must play a fundamental role, and it is therefore important to understand what the learning goals should be, why and what they should be learnt for. It seems important to analyse some aspects that should be considered when defining the mathematics curriculum and teaching necessary to implement this; it will be done later on, trying to raise some working lines for further discussion.

THE MATHEMATICS TEACHER

Whatever the curriculum and teaching, we consider that the responsible for implementing and applying this is the teacher. It is clear that the factor with the greatest impact on improving is the quality of the teacher. Then it is greatly concerned that most of the time a teacher who gets to teach math in a Primary classroom has not been prepared to fulfil this important task.

It is urgent to face this reality. If the education student comes with insufficient education in math and nothing is done to change it, there will be no way to have teachers taking charge of improving learning of mathematics in the country.

Obviously, we are not talking about submitting the future Primary school teacher for a course (or two) of basic math college level, because the contents will not help to complete the necessary foundation for teaching at school level, nor will help to correct the misconceptions that has already acquired. It means having several very well structured courses that allow teachers to learn what they then must teach.

Many times these students, future Primary school teachers, do courses on teaching mathematics, and although they are good courses, they can remain in generalities and can even be misunderstood by ignorance of mathematical topics. Much more valuable would be investing time in which teachers learn mathematics through situations where they experience teaching methods they need to know and that sometimes they have not experienced as students.

From this perspective, it is also necessary to carefully analyse how to choose those who will train teachers, because we will need people who know and can teach math, and who also have classroom teaching experience, if it is not always possible to have trainers who have both requirements. Now that we have new ways of information management, we might have a better chance of solving this problem.

In brief, training math teachers for Primary level is at the heart of the problem and urgently requires a determined effort from the State for breaking the vicious circle in which we are now. So far there are measures that denote a special interest in this direction. It may be necessary to create a specialized

agency in the field and equip it with the necessary resources for the development of a long-term plan with the help of Peruvian and foreign specialists that may be forming new staff of teachers able to meet this challenge ahead.

One factor that can positively influence the duration of this process of change is the math textbook. A text that carefully organizes learning situations, according to the curriculum and more apparent teaching methodologies, will certainly be an important support for a teacher who is still in the process of developing his/her own capabilities. But it must be a text that takes special care in the sequence of activities to generate the necessary mathematical learning and in the right way. Texts are sometimes responding to a major effort to provide rich learning activities that integrate other areas, which represent knowledge of the context of the child. This includes the development of values and attitudes that promote participation and interaction, but which fail to provide the necessary opportunities for the construction of mathematical concepts and procedures. It's like they distract attention from the important topic.

There are other aspects to consider about the influence of teachers in the learning of mathematics: consistency and continuity. The child needs different learning approaches several times and in various ways to consolidate them. However, consistency and continuity is required to achieve the expected learning. Teacher turnover and lack of coordination among them interfere with this. In the same school, sometimes there are not agreements on methods, use of symbols and vocabulary, both from the same teachers who teach math across grade levels, as with teachers from other areas who use them. Many of these problems would be solved by having regular coordination spaces of teachers, initiated by schools and their directors. Also, good practice of writing, archiving and sharing lesson plans help to create a culture of teaching in each school to enrich the work of all and facilitate greater consistency.

THE CURRICULUM OF MATHEMATICS

It is important that the curriculum responds to both the particularities of mathematical notions as to the development of children and adolescents. It is not simply to organize mathematical knowledge, but to do it the way that best facilitates the development of mathematical thinking of the students.

Regarding this, it is important to organize the contents with consistency with what boys and girls are seeing and experiencing. For example, when extending the numeric field in different grades or levels, it is more preferable to use the decimal system boundaries, from 10 to 100, and so on, than referring to 1-digit numbers (up to 9), of 2 digits (up to 99). Children observe their 10 fingers, and from that they collect 10 tens, and so on. Teaching numbers from 1 to 9 in 1st grade is not very consistent with children's experience of counting 1 to 10 with their fingers. Besides, this does not favour the construction of the meaning of decimal number system nor provides the basis of calculation strategies in this system.

Moreover, it is important to organize the items that are related by making these links evident instead of separating them. This applies to, for example, inverse operations, but also the relations that may exist between geometry and arithmetic (area and multiplication), including a multiplication table and its double, or between a function, its graph and the solution of its equation. In the same way, it makes more sense to introduce the real numbers when learning how to solve quadratic equations than long before.

Linking different topics can also help giving more presence to those relegated in the curriculum and that are of the most important and functional for the general population, such as measures, geometry and statistics.

Finally, it is important to organize the curriculum following the principle of spiral learning. There are mathematical concepts and procedures which should be approached several times, each time expanding the knowledge and mastery, instead of grouping them in completely separate sections. For example, it is better to learn the functions, one by one, throughout high school, than learn them all in a single grade.

OPERATIONS IN THE CURRICULUM

The Primary curriculum focuses mainly on learning the operations and their techniques. These learning processes, as well as the actual use that people give in the real context of operations, require the development of various mental calculation strategies and preoperative calculation. It would be important to make this more explicit in the same curriculum.

Moving so quickly to operational techniques brings several drawbacks. First, by not giving enough time before students get to them, their algorithms are not completely understood and are applied mechanically, wasting the opportunity that the area of mathematics must always provide for the development of mathematical thinking . Therefore, it reinforces the false belief that mathematics offers always just a way to work things out and have no room for flexibility and creativity. Finally, these strategies and preoperative mental arithmetic are not learned and, therefore, are wasted in their use in daily lives of children as future citizens.

There are also critical topics in the curriculum, such as fractions that mark the time when many children begin to stop enjoying math; or algebra, which globally has teachers wondering why it causes such difficulty to learn. It would take too long to talk about them but it may suffice to say here that they are issues that involve many nuances and interpretations. Sometimes they are contradictory or go against what students commonly understand. So it is required to have the time to address them calmly and from each of the different perspectives necessary in the curriculum.

TEACHING MATHEMATICS

Despite any organization of the learning in the curriculum, even more important is the choice of teaching principles. They will define the learning experience in the classroom.

For this election, it is important to put forward the question of the meaning of teaching mathematics. The central goal is to teach how to think mathematically. Not only to discover the logic, precision, abstraction and formalization in mathematics but, most especially, to find patterns in the real world that make it useful to use mathematics to understand and intervene in it ("mathematize"). These two sides can complement very well, depending on how they are worked in a math class.

Obviously, we're talking about a math class that does not focus on the mechanical repetition of algorithms and does not measure efficiency by the complexity of

the operations to be solved. This is a math class in which students find meaning of what they do and can build new concepts.

For this to happen, it is convenient to consider several aspects, some of which are mentioned below.

Many times, problems in math classes have been the only way to verify if students can apply what they learn. Actually, they should be most useful in the construction of new concepts. It is from the actual real problems that students can regain their previous knowledge and begin to ask questions about new concepts and their use. Working problems to start new mathematical learning makes this learning almost naturally flow with little intervention from the teacher, who at times, only names them and formalizes some aspects that students discover by themselves. In this way it becomes clear that the patterns of mathematics were already in reality, learning becomes meaningful and is pleasant.

Obviously, we are not talking about a "typical" problem we solve repeatedly or a problem that one reads from a book and solves with the only interest of finding an answer. It is about more open problems that we build (if possible in a real context, with elements from the reality) and discuss together.

To support these processes of construction, it is very useful to use concrete material. Its use allows students to try different strategies, answer questions through trial and error and make what they are thinking visible. This manipulation can sometimes be done virtually with the tools that technologies provide us with today. In both cases, students are expected to verbalise and represent the notions that they are building. Language is also crucial in the development of mathematical thinking.

Repetition is often a learning method to fix what has been learned in the memory, and this is not strange to math class. However, we must consider the right way to use this strategy: something that has not been understood yet should not be repeated mechanically; it should be repeated in relation to other learning and with varied concrete situations; and where possible, the repetitions should include new aspects or nuances of what is to consolidate. In short, we should try to encourage thoughtful repetition.

This variety should be given especially when solving problems. Students and people in general need a wide database when it comes to our experience in the solution of problems. The more problems we solve, the more possibilities to solve new ones we develop.

Games are a good way to achieve this thoughtful repetition while having fun. The issue of motivation and enjoyment of learning mathematics is not so simple, and it is extremely important if we think of the dedication and perseverance required to achieve complex learning goals.

Games are also a powerful tool facing another aspect that teachers should clearly consider: What is the role of errors in the experience of learning mathematics? It is desirable that errors are seen naturally in the learning process and actually as necessary to deepen. A teacher who understands this will try to have a working environment in the classroom to embrace students and give them confidence to work openly and constructively using errors. This situation occurs more spontaneously during playing games.

Finally, I would like to refer to evaluation in math. First, it is important to think of assessment situations beyond checking algorithms and counting correct answers. Describing expected results and strategies that students must bring into play is a different way of assessing from the one normally used by math teachers. The use of rubrics that describe the four levels of achievement (hopefully this approach will also soon move to Secondary) is a practice that is not yet sufficiently diffused. Similarly, giving feedback should be encouraged in both directions (the teacher giving light on the difficulties seen in the performance of their students, students letting their teachers know what they are failing to understand). It can also help promoting students' self-assessment to the extent that they know better what is expected from them to achieve, thus changing evaluation from its punitive character and becoming a real strategy to improve students' learning.

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